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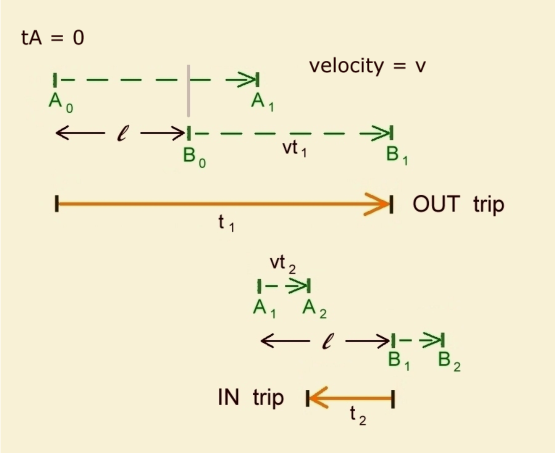
APPENDIX B

As noted on page 86, Einstein's clock synchronization dictates that clock B will have a reading which is less than that of clock A by the amount v, the velocity of the AB system, where those two clocks of the same inertial frame have a rest spatial separation of 1 light second (e.g., 0.8 light second separation for v = 0.6c).

In the first edition of *Relativity Trail*, we omitted the formal demonstration of that fact, as we regarded it as somewhat trivial. Below, we offer that missing step of our Lorentz transformations derivation, noting also, that it's not quite so trivial as we had imagined.

Recall that we are using "1" as the value for c, and that velocity is given as a decimal percentage of the speed of light. Since distance is the product of speed and time, we chart distance on the graph. And since one light-second is the distance light travels in one second, we concern ourselves strictly with numerical values absent of units. Both "t" and "vt" describe distance on the graph. One second universal time = one light second distance.



We proceed much as we did on pages 36-37. Having already determined that $\ell = \mathcal{L} = (1 - v^2)^{1/2}$ on page 37, we note the following:

For the out trip:

Distance traveled by light ray = t_1
where t_1 is universal time.
Distance traveled by AB system = vt_1.

$t_1 = vt_1 + \mathcal{L}$ ==> $\mathcal{L} = t_1 - vt_1$ ==> $\mathcal{L} = t_1(1-v)$
$t_1 = \mathcal{L}/(1-v)$ [eq 1]

For the in trip:

Distance traveled by light ray = t_2
where t_2 is universal time.
Distance traveled by AB system = vt_2.

$t_2 + vt_2 = \mathcal{L}$ ==> $t_2(1+v) = \mathcal{L}$
$t_2 = \mathcal{L}/(1+v)$ [eq 2]

Einstein's clock synchronization requires that tB = 1 and that t'A = 2, so that tA - tB = tB - t'A (i.e., 0 - 1 = 1 - 2) for all v.

B's clock rate is $\mathcal{L}$t. Therefore, from eq 1:

$tB = 1 = \mathcal{L}(\mathcal{L}/(1-v)) + t_iB$ [eq 3]

where t_iB is the initial reading of clock B (when clock A reads 0) as seen against the universal frame.

We wish to know whether indeed $t_iB = -v$.

A's clock rate is of course also $\mathcal{L}$t.
Therefore, from eq 2:

$t'A = 2 = \mathcal{L}(\mathcal{L}/(1+v)) + \mathcal{L}(\mathcal{L}/(1-v))$ [eq 4]

Doubling eq 3 makes it equal to eq 4, therefore:

$2\mathcal{L}(\mathcal{L}/(1-v)) + 2t_iB = \mathcal{L}(\mathcal{L}/(1+v)) + \mathcal{L}(\mathcal{L}/(1-v))$

$2t_iB = \mathcal{L}(\mathcal{L}/(1+v)) + \mathcal{L}(\mathcal{L}/(1-v)) - 2\mathcal{L}(\mathcal{L}/(1-v))$

$2t_iB = \mathcal{L}(\mathcal{L}/(1+v)) - \mathcal{L}(\mathcal{L}/(1-v))$

$2t_iB = (1-v^2)/(1+v) - (1-v^2)/(1-v)$

$2t_iB = [(1-v)(1-v^2) - (1+v)(1-v^2)] / (1+v)(1-v)$

$2t_iB = (2v^3 - 2v) / (1-v^2)$

$2t_iB = -2v$

$t_iB = -v$